

Cover Sheet for Technical Memorandum

The information contained herein is for the use of employees of Bell Laboratories and is not for publication (see GEI 13.9-3)

Title: A Study of PORT Mathematical Subroutine Library Usage

Date: December 19, 1979

FINAL

Other Keywords:

TM: 79-3731-9

Version

Author(s)
Phyllis A. Fox

Location MH 2D-231 Extension 4747

Charging Case: 39199
Filing Case: 39199-9

the file put into

PARTICAL

101110375

t an chined

FOX

ABSTRACT

Various aspects of the current usage of the PORT Mathematical Subroutine Library of Fortran programs are described. First the development of the library and its relationship to other available Fortran mathematical libraries is outlined. Then an accounting is given of the number of current PORT installations both within and outside Bell Labs. Finally a study of PORT usage on the Honeywell computer at Murray Hill is reported. In this study data were gathered on the usage of a selected set of programs, and the results were used to survey usage of individual programs, usage by department and area, and types of numerical computation performed.

Pages Text: 5

Other: 18

Total: 23

No. Figures: 6

No. Tables: 3

No. Refs.: 10



subject: A Study of PORT Mathematical Subroutine Library Usage

Case: 39199 File: 39199-9 date: December 19, 1979

from: Phyllis A. Fox MH 3731 2D-235 x4747

TM: 79-3731-9

MEMORANDUM FOR FILE

1. Introduction

This memorandum summarizes aspects of usage of the PORT Mathematical Subroutine Library of Fortran programs.

The following section provides a general description of the library, including something of its history as well as an overview of its contents, portability, size and so on. The relationship of the library to other Fortran mathematical libraries is also discussed there, especially with regard to the Bell Labs environment. The section ends with an account of the current PORT installations on computers both within and outside Bell Labs.

Section 3 deals specifically with the use of PORT on the Honeywell computer at Murray Hill. Data have been gathered on the usage of a kernel of the more heavily used programs — the names of the routines called and the user identification were recorded. Tables summarizing these data are included.

A final section evaluates the current and potential position of the PORT Library.

2. A General Description of the PORT Library

The PORT Mathematical Subroutine Library of portable programs for numerical computation has been under development at Murray Hill since 1974. The first version of the library was released in the summer of 1975. This edition was a small library of 151 subprograms, many of them lower-level routines without user documentation. However, at this release the portability and all the basic structure of the library was established and implemented.

The library is made portable by two features:

- All programs are written in the portable subset of ANSI Fortran specified by the PFORT verifier [1].
- All programs involving machine-dependent quantities use PORT function subprograms to access the values.

The almost literal portability of the library is attested to by the fact that a single version of the tape is maintained. When a copy of the library is requested, this tape is copied (in the appropriate character format) and sent out together with an installation manual explaining how to activate the machine-dependent constants appropriate to the computer at hand.

These approaches to portability have proved very successful. In fact a recent book on programs for digital signal processing published by the IEEE [2] specifies as a standard for the published programs the PFORT language subset of ANSI Fortran and the use of the PORT functions for obtaining machine-dependent values used in the programs.

PORT is based on an automatic dynamic storage scheme simulated by a stack in named COMMON, and errors are automatically handled. These capabilities permit the calls to PORT programs to be free from parameters for scratch storage or for error flags, so that usage is simplified. More detail on these topics can be found in [3] and [4].

PORT 2

The second (and current) edition of PORT was released in the summer of 1977. This version contains 550 subprograms, of which 125 are documented in user reference sheets. The other subprograms are lower-level modules. There are 41,000 lines of Fortran in the library source. The first printing of the user reference manual for PORT 2 was 700 copies; an additional 400 manuals were printed in July 1978. There are 475 pages of documentation in the manual.

The majority of the programs in PORT 2 have been written by people at Murray Hill. A few programs from the open literature have been adapted to PORT, but always with the permission of their authors. At the moment, no statistical programs are included in the PORT Library.

Relationship of PORT to other program libraries

Of the many numerical libraries that have been developed both in this country and abroad by computer manufacturers, industry, government and universities, the best known ones probably are the IMSL (International Mathematical and Statistical Libraries, Inc.) library [5], and the British NAG (Numerical Algorithms Group) library [6]. Then, aside from statistical libraries which are not discussed here, there are some important special purpose libraries developed by the NATS project [7], funded and staffed by governmental and university groups at Argonne National Laboratory. These include EISPACK [8] containing programs for finding the eigenvalues and eigenvectors of various classes of matrices, FUNPACK [9] which contains programs for computing special functions. LINPACK [10] which is a collection of subroutines for analyzing and solving systems of simultaneous linear algebraic equations, and MINPACK, a package of minimization routines under development. Some of the NATS packages are more portable than others. The programs in EISPACK and in LINPACK can usually be adapted to a specific machine by changing certain machine-dependent constants. FUNPACK, a highly accurate set of routines, is only available in editions hand-tailored to a few large-scale machines: IBM 360-370, CDC 6000-7000, UNIVAC 1108-1110.

EISPACK has been installed on the IBM/Amdahl computers at Holmdel, and on the Honeywell computer at Murray Hill. The IMSL library is also installed at Murray Hill, and sees some use for programs covering areas not available in PORT. However the programs in that library are less robust and not always state-of-the-art. The NAG library would be useful to have, but there has been an impasse since 1974 between our lawyers and theirs.

Incidentally, the IMSL Library costs \$1400/year and NAG between \$1200 and \$1400 per year. PORT is a licensed product with a one-time charge, currently \$3500/year, to industry or government centers, but available to universities for a handling charge of \$100.

Computer systems on which PORT is installed

The PORT Library has been installed on 20 computers within Bell Labs. The first table, Exhibit Ia in the appendix, lists, for each of these computers, the installation date for PORT, the location of the computer and the person in charge. The large-scale computers include the Cray, Honeywell, IBM/Amdahl, IBM 3033, CDC, and UNIVAC. The smaller ones include the PDP 11, VAX, Interdata, Data General Eclipse and Harris.

Exhibit 1b lists the 50 sites outside of the Labs to which PORT has been sent. Of these, 40 are educational institutions, 19 of which are not in the United States. Six governmental/commercial licenses have been signed for PORT — two at the National Center for Atmospheric Research at Boulder, Colorado, one at Xerox Corporation in Webster, New York, one at the International Institute of Applications and System Analysis, Laxenberg, Austria, one at the Telecommunication Laboratory in Taipei, Taiwan, and one at the National Bureau of Standards in Washington, D.C.

Exhibit 2 lists the 37 different machine/system configurations at PORT sites.

3. Usage of the PORT Library on the Honeywell Computer at Murray Hill

Usage of the PORT library has been studied on the Honeywell computer at Murray IIIII. The GCOS operating system on the Honeywell 6000 series of computers keeps a running tally of the number of accesses to a file. Since January of 1976 a monthly check of PORT usage has been recorded by reading these access counts for the PORT library files (both the BCD and the ASCII versions). This usage, translated into accesses per day, is shown in a graph, Exhibit 3, in the appendix. The number of uses per day has been obtained by dividing total use by the number of days in the month, not simply working days, because a good deal of computing occurs over weekends. The curve is rather jagged, but various trends are discernible. There was a steady climb in usage up to a peak early in 1977. Then a decline set in, some of which was due to the installation of PORT on a number of smaller computers within the Labs. Since people tend to keep on using programs to which they have grown accustomed, we presume that usage was merely transerred to the smaller machines, but we have no way of measuring actual usage elsewhere. Also, and again there is no way to check the fact, many people tend to run programs directly from load modules (H* files in GCOS terminology) into which library subroutines have been bound. Such usage is not monitored by our procedures. During 1978 usage seemed to level off at about 150 accesses per day to the library, but in the summer of 1979 it grew to 200 accesses per day.

Earlier this year a project was undertaken to monitor the PORT library on the Honeywell computer in somewhat more detail, by finding out, for a selected set of programs, how often they were used and by whom. In each selected program a trap with a first-time switch was used to escape to a recording program, and a file of usage statistics was accumulated. The user id and the name of the called program were obtained and tallied. Our experience in the monitoring process and a discussion of the data obtained are reported below.

Programs monitored

Not all 525 subprograms in the library were monitored; rather a subset was selected by the principal contributors of PORT programs, J. L. Blue, P. A. Fox, N. L. Schryer and D. D. Warner. The selection process was based on the modular structure of the library: in many cases PORT programs which are documented and seen by the user represent the upper level of a multilevel calling structure. In such a case only the lowest-level subprogram was monitored. Exhibit 4a in the appendix lists the names of the monitored subprograms, and, in the case of a lower-level routine, also the names of the documented higher level calling routines.

Exhibit 4b shows the grouping of the various monitored subprograms into general areas of numerical computation. Sometimes it is not possible to know if a given routine was used for say interpolation or quadrature, but the grouping represents the most probable partitioning.

Gathering the data

The data discussed below were gathered during the months of March, April, May and June 1979. However, since usage of the library tends to vary over time, any set period of monitoring can be misleading. Often one or two particular users call heavily upon a set of PORT routines for a certain time period and then depart to write up their results, and another group of people and programs become active.

The reader should be aware that the data represent only the usage of available programs; there is a very strong demand for programs covering areas such as least-squares fitting, optimization, solution of stiff ordinary differential equations, solution of partial differential equations, etc., which are not covered in PORT. Some of these needs are met by packages built over the PORT library, such as the excellent partial differential equation solver POST, but an effort must be made to fill the gaps in PORT.

What programs are used?

During the period under study a total of 9736 calls were made to the monitored programs in the PORT Library. The names of the programs called and the number of calls to each are listed in Table 1, sorted by most used first.

Before discussing these data, a few caveats should be made. As noted in the table, not all double-precision versions were monitored. In the case of the Fast Fourier Transform subprogram, FFT, and the quadrature program, BQUAD, the authors were interested initially only in seeing if there was some usage in these areas. Particularly for FFT's, the bulk of the use at the Labs (and there is a lot) is with individually tailored routines or special purpose use on dedicated smaller computers. Another caveat is in order on the monitoring of the trigonometric functions: if a user who uses PORT calls a trig function, the function will be obtained from the PORT library which is searched before the system library, so the counts for trig functions are not necessarily PORT-related.

The areas of numerical representation represented by the subprogram calls given in Table 1 are discussed under *Patterns of use* below.

Who uses PORT and how much

Data accumulated during the four spring months provided a list of 86 individual users of the PORT library on the Honeywell computer over that period. More than 60% of the users came from Area 10, and 17% from area 20; the rest of the usage came mostly from Areas 30 and 40. Now if we consider these users vis-à-vis the number of calls to PORT made by people in the various areas, we find that 72.7% of the usage came from Area 10, some 14% from Area 20, and the rest from Areas 30 and 40. These data are shown in Table 2. (One user from Area 60 is not included in the tabular data.)

Patterns of use

Since the monitoring process provided information both on the location (Area) of the user and the name of the called subprogram, it was possible to study the particular type of numerical computation being carried out, using PORT, by users in each Bell Labs Area during the monitored period. The subprograms were grouped by computational type (as given in Exhibit 4b), and then the number of calls to each subprogram within each given category were summed. The ratio of the sum of calls in each category to the overall number of calls is the percentage of use shown in the last column of Table 3. To find the use within Areas 10, 20, 30 and 40, the same procedure was applied within these groupings. These percentages are shown in the corresponding columns of Table 3.

Quadrature is clearly the winner, with zero-finding a strong second. Calculation with splines is a growing sector. It may be a misrepresentation, in fact, to separate the categories of quadrature and splines — in a certain sense both of these areas fall under "approximation."

During the period in question, the large number of programs in linear algebra which are to be included in PORT were in use but not monitored. The figure given in the tables represents use of the only two subprograms in this area in PORT 2. Also the category of eigenvalue-eigenvector computation, as represented in the table, is not representative of real use in this area; the EISPACK package of subprograms provides a very good general source of programs for serious eigenvalue-finding programmers.

One of the surprising statistics in Table 3 is the relatively low use of ordinary differential equation programs. When the first edition of PORT was being put together in 1974 there was a great deal of ODE computation being done. Now the major interest is in partial differential equation packages, one-dimensional as in POST, and going into two-dimensional developments.

Yet another caveat on the data: the category of optimization as included in the tables represents only the program FMIN to find a single minimum of a single function between two given points. PORT 2 does not satisfy the current demand for all kinds of optimization — constrained and unconstrained, least-squares or general linear or nonlinear, with or without derivatives provided by the user. We hope to be able to provide programs covering most of these areas.

4. Summary

In this first summary report on usage of the PORT library (again, only for the Honeywell computer—we have no access to patterns of use of the library at other sites), we have noted that the most heavily used programs are in the numerical areas of quadrature and spline approximation. This is not surprising because these are the two areas in which PORT has been strongest; the programs are exceptionally good and have been in use for many years. We will soon add to the library the almost 200 high-caliber programs written by Linda Kaufman for the solution of linear equations. There are programs for general, banded, symmetric, and sparse systems. Future surveys undoubtedly will show heavy use of these programs. From inquiries we also know that there is a demand for linear and nonlinear programming routines, for least-squares approximation of various sorts, for stiff differential equation solvers, and for various routines for the solution of partial differential equations. All these areas are under study and will give rise to useful programs. Finally there is the question of statistical routines. We have been reluctant to get into this area for a number of reasons. We know there is a demand for portable programs in standard areas of statistical computation, but this may fall outside of PORT's domain. We welcome suggestions.

5. Acknowledgments

Almost all of the programs in PORT have been contributed by James L. Blue, Phyllis A. Fox, Linda C. Kaufman, Norman L. Schryer and Daniel D. Warner. Much of the framework of PORT, the parts dealing with error handling and storage management were designed by Andrew D. Hall, and both he and W. Stanley Brown have made further valuable suggestions improving the course of PORT's development. A. D. Hall also provided a crucial program used in the actual monitoring process. Finally, the sifting and studying of the collected data was facilitated by the database inquiry language Q being developed by Alfred A. Aho and Brian W. Kernighan.

MH-3731-PAF

Phyllis A. Fox

Mylla Q. Fx

References

- [1] Ryder, B. G. The PFORT verifier. Software Practice and Experience 4, 1974, 359-377.
- [2] Programs for Digital Signal Processing, edited by the Digital Signal Processing Committee, IEEE Acoustics, Speech, and Signal Processing Society, IEEE Press, New York, 1979.
- [3] Fox, P. A., Hall, A. D., and Schryer, N. L. The PORT Mathematical Subroutine Library, Computing Science Technical Report #47, Bell Laboratories, May 1977. (The same as [4] except that Report #47 includes the full listing of the programs for PORT's machine constants, error handling and stack management.)
- [4] Fox, P. A., Hall, A. D., and Schryer, N. L. The PORT Mathematical Subroutine Library, ACM Trans. Math. Software 4, 2, June 1978, 104-126.
- [5] Johnson, O. G., IMSL's ideas on subroutine library problems. SIGNUM Newsletter 6, Number 3, 1971, 10-12.
- [6] Ford, B., and Sayers, D., Developing a single numerical algorithms library for different machine ranges, ACM Trans. Math. Software 2, 1976, 115-131.
- [7] Boyle, J. M., Cody, W. J., Cowell, W. R., Garbow, B. S., Ikebe, Y., Moler, C. B., and Smith, B. T. NATS, a collaborative effort to certify and disseminate mathematical software. *Proc.* 1972 ACM Annual Conf., Vol. 11, pp. 630-635.
- [8] Smith, B. T., Boyle, J. M., Dongarra, J. J., Garbow, B. S., Ikebe, Y., Klema, V. C., and Moler, C. B. Matrix Eigensystem Routines EISPACK Guide, Second Edition, Springer-Verlag, New York, 1976. (See also EISPACK Guide Extension, Springer-Verlag 1977.)
- [9] Cody, W. J., The FUNPACK package of special function subroutines. *ACM Trans. Math. Software* 1, 1975, 13-25.
- [10] Dongarra, J. J., Moler, C. B., Bunch, J. R., and Stewart, G. W., *LINPACK Users' Guide*, Society for Industrial and Applied Mathematics, Philadelphia, Pennsylvania, 1979.

APPENDIX

Exhibit 1a: PORT installations within Bell Labs

Exhibit 1b: PORT installations outside Bell Labs

Exhibit 2: Computer systems represented by PORT users

Exhibit 3: PORT Library - graph of accesses per day

Exhibit 4a: PORT programs monitored on the Honeywell computer

Exhibit 4b: PORT programs grouped by computational type

Table 1: PORT programs: number of calls

Table 2: PORT Library: distribution by Area of amount of use

Table 3: PORT Library: type of numerical computation performed in

different Bell Labs Areas

EXHIBIT 1a

PORT installations within Bell Labs

Installation Date	Computer	Org/Loc	Person in charge		
8/75	Honeywell	373/MH	Phyllis Fox MH X4747 1D-231		
4/76	IBM 370	372/HO	David Muir HO X7640 2G-432		
6/76	CDC Cyber	423/MV	B. F. G. Fink MV X6325 1D-51		
6/76	PDP 11/70	127/MH	M. D. McIlroy MH X6050 2C-526		
8/76	UNIVAC 1110	373/MH	Arthur G. Gross MH X5331 5A-133		
9/76	Eclipse	122/MH	Carolyn Schmidt MII X5342 2D-517		
12/76	Harris S220	153/MH	T. A. Weber MH X2249 1A-364		
7/77	UNIVAC 1108	622/WH	H. T. Balch WH X2669 4A-302		
8/77	Harris /7	227/MH	B. R. Chawla MH X3555 2B-414		
11/77	CDC	439/110	P. Balaban HO X3249 3F-224		
12/77	Interdata	127/MH	A. G. Fraser MH X3685 2C-561		
3/78	Harris /7	227/MH	B. R. Chawla MH X3555 2B-414		
9/78	PDP 11/45	113/MH	M. F. Robbins MH X3325 1E-428		
9/78	PDP 11/45	152/MH	B. C. Wonsiewicz MH X3751 1A-121		
9/78	VAX 11/780	135/HO	T. B. London HO X2006 4E-624		
1/79	Amdahl 474	376/IH	Anthony J. Gaseor HO X3780 1C-235		
1/79	Eclipse	211/MH	D. McE. Boulin MH X3103 2A-240		
7/79	IBM 3033	378/PY	R. L. Rosen PY X6715 2A-225		
10/79	VAX 11/780	211/MH	Wolfgang Fichtner X7628 2A-325		
1/80	Cray-1	373/MH	Phyllis Fox M11 X4747 2D-235		

EXHIBIT 1b

PORT installations outside Bell Labs

PORT 1

Type G = Government E = Educational C = Commercial F = Foreign	Date	Location/ Computer
E	5/1/76	Johns Hopkins Univ., Baltimore, MD PDP 10/Kl
Е	5/1/76	U. of Pittsburgh, Pittsburgh, PA IBM 370/158
E	7/1/76	Millersville State College, Millersville, PA UNIVAC 70
Е	9/1/76	Clemson Univ., Clemson, SC IBM 370/158
EF	9/1/76	U. of Toronto, Toronto. Ont. CANADA IBM 370/165
EF	9/1/76	Laurentian Univ., Sudbury, Ont. CANADA IBM 360/40
Е	10/1/76	Clarkson College, Postdam, NY IBM 360/65
E	1/1/77	California Polytechnic State Univ., San Luis Obispo, CA PDP 11/45 (19 branches of the university)
E	1/1/77	Georgia Inst. of Tech., Atlanta, GA NOVA-830, NOVA-820
EF	2/1/77	The American Univ. in Cairo, EGYPT Century 201

EF	2/1/77	Univ. of Salzburg, AUSTRIA IBM 370/145
EF	3/1/77	Academisch Computercentrum, Utrecht, NETHERLANDS CYBER 73
EF	5/1/77	Katholieke Univ. Leuven, BELGIUM IBM 370/158
E	5/1/77	Vassar College, Poughkeepsie, NY

PORT 2

Туре	Date	Location/ Computer
EF	12/1/77	Heriot-Watt U., Edinburgh, SCOTLAND Burroughs B1726
EF	1/1/78	U. of Alberta, Edmonton, CANADA Amdahl 470v/6
E	2/1/78	U. of North Carolina, Chapel Hill, NC IBM 360/75 and 360/155
EF	2/1/78	U. of British Columbia. Vancouver, CANADA IBM 370/168
EF	2/1/78	Bar-Ilan U., Ramat-Gan, ISRAEL IBM 370/168
E	5/1/78	City College of City Univ., New York, NY PDP 10/KA
E	5/27/78	Johns Hopkins Univ., Baltimore, MD PDP 10/KI
E	6/1/78	Harvard Univ., Sch. of Public Health, Boston, MA PDP 11/70 and PDP 10
E	7/1/78	Kansas State Univ., Manhattan, KS PDP 11/34
EF	7/1/78	U. of Bristol, Bristol, ENGLAND PDP 11/10
E	7/27/78	Georgia Inst. of Tech., Atlanta, GA NOVA-830, NOVA-820
EF	8/1/78	Laboratoire D'Automatique de Grenoble, FRANCE IBM 360
E	8/1/78	Manhattan College, Riverdale, NY PDP 11/70
EF	8/1/78	U. of Auckland, NEW ZEALAND PRIME P400, Burroughs B6714

EF	11/1/78	U. of Wollongong, Wollongong, AUSTRALIA INTERDATA
EF	11/1/78	U. of New England, Armidate NSW, AUSTRALIA ICL 1904A, PDP 11/34, DEC2050
E	11/1/78	lowa State U. of Science and Technology, Ames, IA VAX 11/780
G	11/7/78	Argonne National Laboratory (study only) IBM
G	11/26/78	Sandia Labs., Appl. Math Div., Albuquerque, NM CDC 6600, CYBER 176
E	12/1/78	U. of Minnesota, Minneapolis, MN Honeywell 66
EF	12/1/78	U. of Western Ontario, London, Ont. CANADA PDP 10
EF	12/1/78	Capricornia Inst. of Adv. Educ., Queensland, AUSTRALIA Hewlett Packard 3000 Series II
G	1/24/79	NCAR (2 copies), Boulder, CO CRAY, CDC 7600
	3/8/79	Western Electric Engineering Research, Princeton unspecified computer
E	4/1/79	Duke Univ., Durham, NC PDP 11/70
E	4/1/79	U. of Texas System Cancer Center, Houston, TX CDC CYBER 171
С	6/18/79	Xerox Corp., Webster, NY unspecified computer
CF	6/18/79	Int. Inst. of Applic. System Analysis, Laxenberg, AUSTRIA unspecified computer
E	7/16/79	California Institute of Technology, Pasadena. CA IBM 3032
E	8/2/79	University of California, San Francisco, CA IBM 370/148 and PDP 11/70

. .

E	8/9/79	Purdue University, West Lafayette, Indiana PDP 11/70
EF	9/10/79	Regionales Rechenzentrum Erlangen, GERMANY CYBER 172 and TR445
EF		Australian National University, Canberra, AUSTRALIA PDP-10 (KA-10 processor)
CF	9/20/79	Telecommunication Labs, Taipei, Taiwan unspecified computer
G	10/4/79	National Bureau of Standards, MD unspecified computer
E	10/12/79	Rockefeller University, New York, NY PDP 11/70

EXHIBIT 2

Computer systems represented by PORT users

BURROUGHS B6700	DEC VAX 11/780
CDC 6600	HEWLITT PACKARD 3000
CDC 7600	HONEYWELL 66
CDC CYBER 171	IBM 3032
CDC CYBER 172	IBM 360
CDC CYBER 176	IBM 360/40
CDC CYBER 73	IBM 360/65
CENTURY 201	IBM 370/125
CRAY-1	IBM 370/145
DATA GENERAL NOVA-820	IBM 370/148
DATA GENERAL NOVA-830	IBM 370/158
DEC 2050	IBM 370/165
DEC PDP 10	IBM 370/168
DEC PDP 10/KA	ICL 1904A
DEC PDP 10/KI	INTERDATA 8/32
DEC PDP 11/10	PRIME P400
DEC PDP 11/34	TELEFUNKEN TR445
DEC PDP 11/45	UNIVAC 70
DEC PDP 11/70	

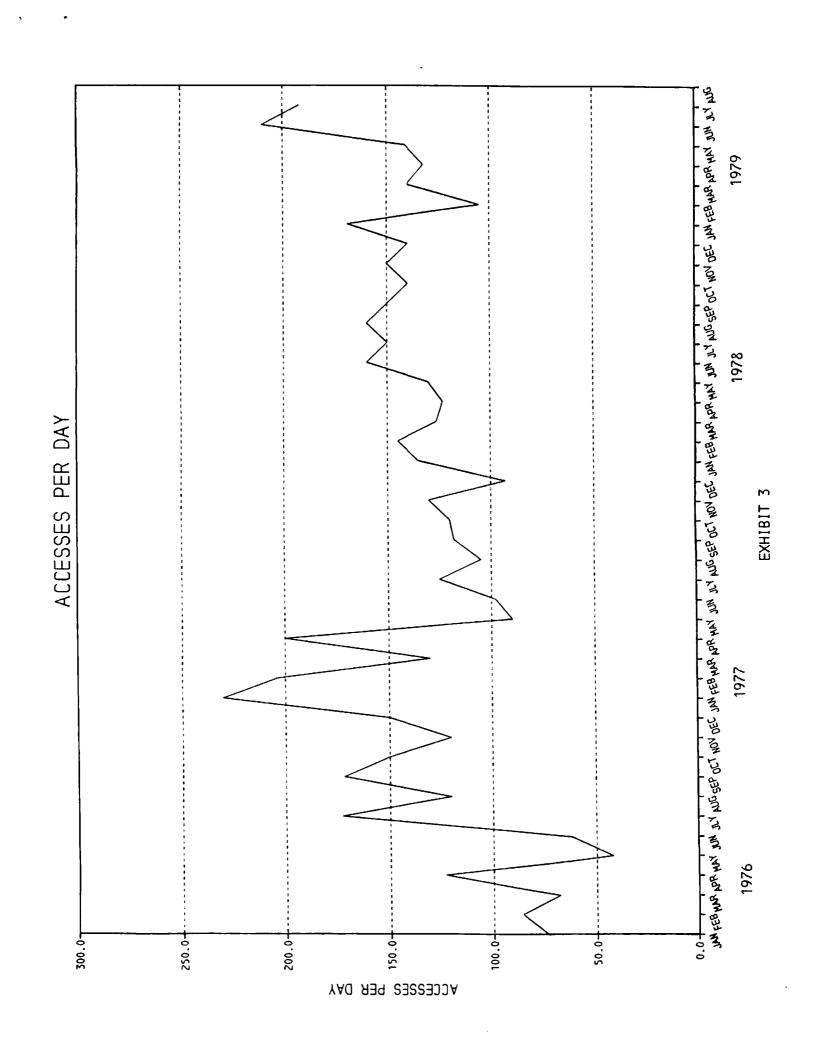


EXHIBIT 4a

PORT programs monitored on the Honeywell computer

```
A3PLNN
  called by:
            BSPL1 - basis spline evaluation with selected derivatives
            BSPLD - basis spline evaluation, with derivatives
            BSPLI - integrate basis splines
            BSPLN - basis spline evaluation
            EEBSI - estimate B-spline error over selected intervals
            EESFI - find absolute error in B-spine fit over selected intervals
            SPLN1 - spline and selected derivatives evaluation
            SPLN2 - tuned spline evaluation
            SPLND - spline and derivatives evaluation
            SPLNE - spline evaluation
            SPLNI - spline integration
(D3PLNN double-precision version of A3PLNN)
BISLR
   called by:
            BESRI - Modified Bessel function, I, real argument, integer order
            BESRJ - Bessel function, J, real argument, integer order
BQUAD
             - integrate piecewise-smooth function
BURMI - best uniform rational approximation (given an initial one)
   called by:
            BURAM - best uniform rational approximation on a mesh
(DBURM1 double-precision version of BURM1)
CISPET
   called by:
            CSPDI - cubic spline differentiation
            CSPFI - cubic spline fit
            CSPIN - cubic spline interpolation
            CSPQU - cubic spline quadrature
            double-precision version of C1SPFT)
(DC1SPF
C2SPFT
   called by:
            CSPDI - cubic spline differentiation
            CSPFE - cubic spline evaluation
            CSPIN - cubic spline interpolation
```

(DC2SPF double-precision version of C2SPFT) C2SPQU called by: CSPQU - cubic spline quadrature (DC2SPQ double-precision version of C2SPQU) EIGEN - eigenvalues and eigenvectors of a general real matrix (DEIGEN double-precision version of EIGEN) FFT - Singleton's mixed radix Fast Fourier Transform called by: - Fast Fourier Transform (complex data) FFTC1 - inverse Fast Fourier Transform (complex data) FFTR - Fast Fourier Transform (real data) FFTRI - inverse Fast Fourier Transform (real data) FMIN - local minimum (one dimension) (DFMIN double-precision version of FMIN) GAUSQ - abscissae and weights for Gauss quadrature called by: GQ0IN - Gauss-Laguerre quadrature GQM11 - Gauss-Legendre quadrature L2SFF - B-spline fit to a function L2SFH - B-spline fit to the derivatives of a function (DGAUSQ double-precision version of GAUSQ) LPPH2 called by: BURM1 - best uniform rational approximation (DLPPH2 double-precision version of LPPH2) LST2D called by: LINEQ - solution of linear equations LSTSQ - least-squares solution of linear equations

(DLST2D) double-precision version of LST2D)

ODES2

called by:

ODEQ - integrate a set of integrals

ODES - ordinary differential equation solution ODES1 - ordinary differential equation solution

(DODES2 double-precision version of ODES)

RIQINT

called by:

QUAD - integrate a function, absolute error RQUAD - integrate a function, relative error

(D1QINT double-precision version of R1QINT)

UNI - uniform random deviate

ZIONE

called by:

ZONE - zeroes of nonlinear equations

ZONEJ - zeroes of nonlinear equations with Jacobian provided

(DZIONE double-precision version of ZIONE)

ZERO - single real root

(DZERO double-precision version of ZERO)

Trigonometric routines:

ACOSH

ARCOS

ARSIN

ASINH

ATANH

COSII

DACOSH

DARSIN

DASINH

DCOSH

DSINH

DTANH

SINH

TAN

TANH

EXHIBIT 4b

Monitored programs grouped by computational type

(Double-precision names are not listed)

Splines - including approximation, evaluation, differentiation

A3PLNN C2SPFT

Quadrature

BQUAD CISPFT C2SPQU GAUSQ R1QINT

Trigonometric and special functions

trigonometric functions as listed in Exhibit 4a B1SLR

Zero finding

ZERO ZIONE

Linear equation solution and least-squares fitting

LST2D

Differential equation solution

ODES2

Fast Fourier transform

FFT

Eigenvalue and eigenvector problems

EIGEN

Optimization

FMIN

Random number generation

UNI

Rational approximation

BURM1 LPPH2

TABLE 1
PORT programs: number of calls

Single-precision		Double-p	Total	
Name Calls		Name	Calls	Calls
rlqint a3plnn zero zlone lst2d gausq odes2 clspft asinh arsin c2spqu tan c2spft fft eigen arcos fmin uni burml lpph2 cosh b1slr sinh atanh bquad tanh	492 212 1014 907 619 141 383 332 0 101 170 205 203 201 152 138 87 75 0 0 52 36 14 24 21 5	dlqint d3plnn dzero dzlone dlst2d dgausq dodes2 dclspf dasinh darsin dc2spq dc2spf deigen dfmin (none) dburml dlpph2 dcosh dsinh dtanh	1136 1173 215 63 54 495 171 68 392 156 68 0 2 0 66 64 5	1628 1385 1229 970 673 636 554 400 392 257 238 205 203 201 154 138 87 75 66 64 57 36 30 24 21 13
				9736

^{*}Double-precision version not monitored.

TABLE 2

PORT Library

Distribution by Area

of number of users and number of calls to subprograms

Area	Number of users	Percentage of users	Number of calls	Percentage of calls
10	53	61.6	7077	72.7
20	15	17.4	1409	14.4
30	7	8.1	711	7.3
40	10	11.6	334	3.4

PORT Library

Type of numerical computation

performed in different Bell Labs Areas

TABLE 3

	Percentage of calls					
Туре	Area 10	Area 20	Area 30	Area 40	Overall	
Quadrature	35,5	18.8	2.4	.6	30.	
Zero finding	29.8	5.3	1.8	1.5	22.6	
Splines	12.3	37.8	10.8	18.6	16.3	
Trig and Bessel	6.1	21.6	46.3	18.6	11.8	
Linear algebra/least-squares	6.7	2.6	0	49.4	6.9	
Differential equations	5.7	3.8	13.5	0	5.7	
FFT	.6	5.8	10,4	.6	2.	
Eigenproblem	.4	0	13.2	10.2	1.6	
Rational Approximation	1.6	1.0	0	6.3	1.3	
Optimization	.4	3.3	.6	0	.9	
Random numbers	.9	0	1.0	0	.8	
	100	100	100	100	100	

TM SUBMITTAL FORM (See Instructions on Other Side)

Correspondence Files HO 5C-101, X6911 Please arrange for checking, duplication, addressing, and distribution of the attached TM manuscript. Identification Contact Person (if other than the first author): (Name, Location, Room, Extension) PORT Mathematical Subroutine 79-3731-9 TM Number(s): Print on D both sides D one side only. Assembly Sequence (see instructions on other side) Cover sheet (leave this line blank):___ Body: 1-6, Appendix, Exhibits la, 16 (5 pages) 2, 3, 4a (3 pages), 4b (2 pages), Tables 1, 2, 3 Substitute Pages for External Version (see instructions on other side) Extra Dunlication (see instructions on other side) ____External Cover Sheets:_____ TM Cover Sheets: _ Complete TMs:__ _External Papers:__ Approvals (see instructions on other side) Author(s):__

Form N-9050 (8/77)

Director(s):